a) A linear combination of a set of vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ is an ~~ ~~ vector of the form

(hi 5060)

$$\vec{v} = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \dots \alpha_n \vec{v}_n$$

example $\vec{v} = 3\hat{i} + 4\hat{j} + \hat{k}$

$$\vec{v}_2 = 3\hat{i} + 4\hat{i} + \hat{k}$$

b) A set of vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ is lineasly independent if none of the vectors can be expressed in terms of the other ones

Example

$\vec{v}_1 = \hat{i}$

$\vec{v}_2 = \hat{i} + \hat{j}$

$\vec{v}_3 = \hat{i} + \hat{j} + \hat{k}$

} linearly independent

{ basis of 3D space just lika $\hat{i}, \hat{j}, \hat{k}$

Example

$\vec{v}_1 = \hat{\imath}$

$\vec{v}_2 = \hat{\imath} + \hat{\jmath}$

$\vec{v}_3 = \hat{\imath} - \hat{\jmath}$

$\vec{v}_1 = (\vec{v}_2 + \vec{v}_3)/2$

linearly dependent

Simplest way to check linear (in)dependence:

Try to create zero from the vectors with not all vectors multiplied by zero → dependent

$$\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \cdots + \alpha_n \vec{v}_n = 0$$

only if $\alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$

$\rightarrow$ linearly independent

otherwise linearly dependent

Consequence

You can have at most $\overset{n}{3}$ independent vectors

in $\overset{n}{3}$ dimensional space

$$\underbrace{\hat{i}, \hat{j}, \hat{k}} \quad \underbrace{\hat{i}+\hat{j}+\hat{k}}$$

$$\underbrace{\hat{i}+\hat{j}+\hat{k}}_{v_4} = 1\underbrace{\hat{i}}_{\vec{v}_1} + 1\underbrace{\hat{j}}_{\vec{v}_2} + 1\underbrace{\hat{k}}_{\vec{v}_3}$$

$$0 = 1\vec{v}_1 + 1\vec{v}_2 + 1\vec{v}_3 - \vec{v}_4$$

A change of basis is switching to a different basis

e.g. solid body rotations

$\hat{i}, \hat{j}, \hat{k} \rightarrow$ principal axes $\hat{i}', \hat{j}', \hat{k}'$

"vector space" =

the set of all vectors

"spanned" by $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$

= all vectors of the form

$$\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \cdots + \alpha_n \vec{v}_n$$

Matrix of a system of equations

$$1x_1 + 2x_2 - x_3 + 4x_4 = 0$$

$$3x_1 - 4x_2 + 2x_3 - 6x_4 = 1$$

$$x_1 - 3x_2 - 2x_3 + x_4 = 2$$

Matrix notation

$$\begin{array}{cc} & \begin{array}{cccc} j=1 & j=2 & j=3 & j=4 \end{array} \\ \begin{array}{c} i=1 \\ i=2 \\ i=3 \end{array} & \begin{pmatrix} 1 & 2 & -1 & 4 \\ 3 & -4 & 2 & -6 \\ 1 & -3 & -2 & 1 \end{pmatrix} \end{array} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

matrix "A" $\qquad \vec{x} = \vec{b} \qquad A\vec{x} = \vec{b}$

matrix multiplication is row-column

General An $m \times n$ matrix is a table of m rows and n columns

Square matrix $m = n$

Indices i and j entry/coefficient

Element in row i and column j is called a_{ij} or Matlab $A_{ij} = A(i,j)$

equality of matrix

requires $m_1 = m_2$ $\quad n_1 = n_2$

and every element is the same

Addition of matrices

$$\overset{A}{\begin{pmatrix} 1 & 2 & -3 \\ 4 & 0 & 2 \end{pmatrix}} + \overset{B}{\begin{pmatrix} -1 & 6 & 3 \\ 8 & 12 & 14 \end{pmatrix}} = \overset{C}{\begin{pmatrix} 1-1 & 2+6 & -3+3 \\ 4+8 & 0+12 & 2+14 \end{pmatrix}}$$

$$= \begin{pmatrix} 0 & 8 & 0 \\ 12 & 12 & 16 \end{pmatrix}$$

Multiplication by a scalar

$3 \quad A \quad = B$

$$3\begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 1 & 4 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 0 \\ 3 & 12 \\ 6 & 18 \end{pmatrix}$$

Matrix multiplication

$$\overset{A}{\begin{pmatrix} 1 & 1 & 2 & 1 \\ 4 & 1 & 6 & 2 \end{pmatrix}} \overset{B}{\begin{bmatrix} -1 & 8 \\ 2 & 1 \\ 1 & 1 \\ 12 & 6 \end{bmatrix}} = C$$

$$= \begin{pmatrix} 15 & 17 \\ 28 & 51 \end{pmatrix}$$