hi 5060

Linear algebra

Vectors : for geometry
straight lines
surfaces

Dot product

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$\rightarrow$ number : scalar

Cross product

$\hat{i}\begin{vmatrix} a_y & b_z \\ b_y & b_z \end{vmatrix} - \hat{j}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} =$$

$$\hat{i}(a_y b_z - a_z b_y) + \hat{j}(a_z b_x - a_x b_z) + \hat{k}(a_x b_y - a_y b_x)$$

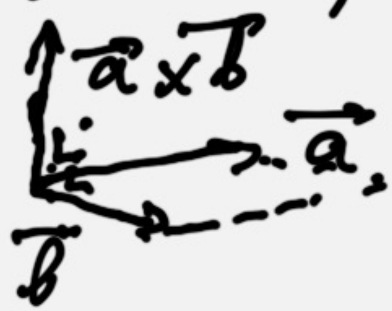
$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(\theta)$

$\vec{a}$ and $\vec{b}$ are orthogonal
if $\vec{a} \cdot \vec{b} = 0$ (definition)

$\vec{a} \cdot \vec{a} = |\vec{a}|^2$

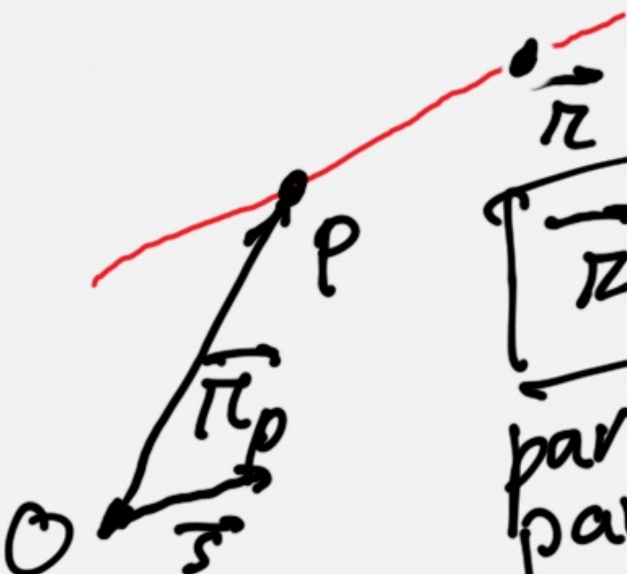
Cross product (vectorial product)

$\vec{a} \times \vec{b} \perp \vec{a}$ and $\perp \vec{b}$



$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

line throug point P parallel to a vector $\vec{s}$

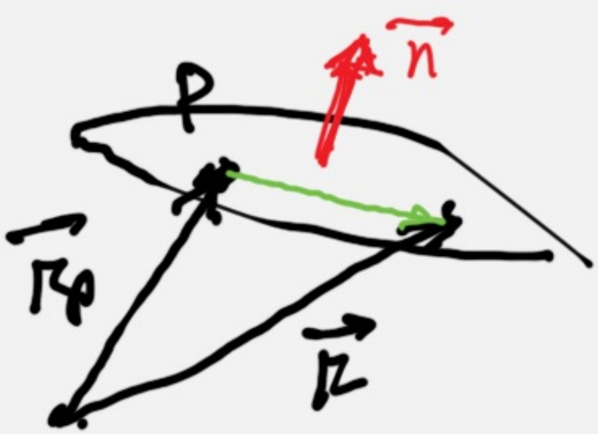


$$\boxed{\vec{r} = \vec{r}_p + \lambda \vec{s}}$$

parametric form

parametric

Plane through a point P normal to a vector $\vec{n}$.



$$(\vec{r}-\vec{r}_p)\cdot\vec{n}=0$$

$$\vec{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$n_x x + n_y y + n_z z = \underbrace{n_x x_p + n_y y_p + n_z z_p}_{\text{known number}}$$

in 3D:

1 eq → plane

2 eq → line

$$\underline{\vec{r} = \vec{r}_p + \lambda \vec{s}}$$

$\left.\right\}$ 3 scalar equations

→ eliminate λ

→ 2 scalar equations

Asked line through $P = (2,-3,5)$ and parallel to the line ℓ

$$\begin{cases} x - y + 2z + 4 = 0 & (1) \\ 2x + 3y + 6z - 12 = 0 & (2) \end{cases}$$

Solution: $\vec{r} = \vec{r}_p + \lambda \vec{s}$

$(1): \vec{n}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

$(2): \vec{n}_2 = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$

$$\vec{s} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(-12) + \hat{j}(4-6) + \hat{k}(3+2)$$

$$= -12\hat{i} - 2\hat{j} + 5\hat{k} = \begin{pmatrix} -12 \\ -2 \\ 5 \end{pmatrix}$$

$$\vec{r} = \vec{r}_p + \lambda \vec{s}$$

(1) asked (2) asked

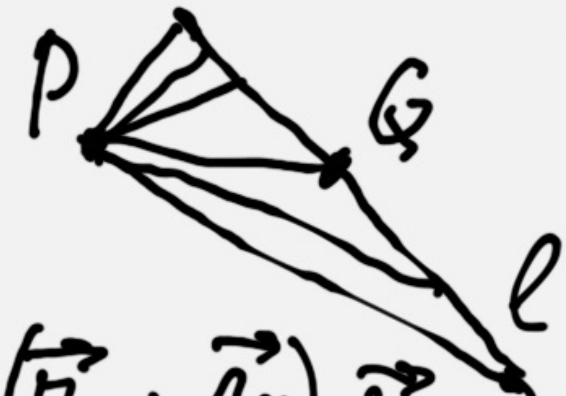
$$\vec{r} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -12 \\ -2 \\ 5 \end{pmatrix}$$

$$x = 2 - 12\lambda$$
$$y = -3 - 2\lambda$$
$$z = 5 + 5\lambda$$

$$\lambda = \frac{2-x}{12} = \frac{y+3}{-2} = \frac{z-5}{5}$$

Asked: the plane through $P=(2,-3,2)$ and the line l

$$l: \begin{cases} 6x+4y+3z+5=0 \\ 2x+y+z-2=0 \end{cases}$$



Solution: need vector normal to the asked plane $\to$ get 2 vectors inside the plane and cross them to get $\vec{n}$

$$(\vec{r}-\vec{r}_p)\cdot\vec{n}=0$$

First vector: take $\vec{n}_1 \times \vec{n}_2$

$\vec{n}_1 = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix}$ $\vec{n}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $\vec{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 3 \\ 2 & 1 & 1 \end{vmatrix}$

$= \hat{i} - 2\hat{k} = \vec{s}$ inside the plane

$x_Q = 0$

$$\begin{array}{rl} 4y + 3z + 5 = 0 & \Big|\; 1 \\ y + z - 2 = 0 & \Big|\; -4 \\ \hline -z + 13 = 0 & \end{array}$$

$z_Q = 13$

$y_Q = -11$ $\quad \vec{t} = \vec{r}_Q - \vec{r}_P$

$\vec{t} \times \vec{s} = \vec{n}$