hi 5060 l'Hôpital.

$\lim_{x \to a} \frac{f_1(x)}{f_2(x)}$ <u>must</u> be ratio

must be indeterminate at a

$$\frac{f_1(a)}{f_2(a)} = \begin{cases} 0/0 \\ \pm\infty / \pm\infty \end{cases}$$

then: $\lim \ldots = \lim_{x \to a} \frac{f_1'(x)}{f_2'(x)}$

$$\lim_{x\to 0} \frac{x - \arcsin(x)}{\sin^3(x)} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$= \lim_{x\to 0} \frac{1 - \frac{1}{\sqrt{1-x^2}}}{3\sin^2 x \cos x} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$= \lim_{x=0} \frac{-\frac{1}{2}(1-x^2)^{-3/2}\, 2x}{6\sin x\cos^2 x - 3\sin^3 x} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$= \lim_{x=0} \frac{-(1-x^2)^{-3/2} - \frac{3}{2}(1-x^2)^{-5/2}\, 2x^2}{6\cos^3 x - 6\sin^2 x - 9\sin^2 x\cos x} \quad \begin{matrix} \to -1 \\ \to 6 \end{matrix}$$

$$= -1/6$$

Total differentials:

error estimates

changes in compound quantities

$$u = u(x, y, z, t)$$

$$a_x = \left(\frac{du}{dt}\right)_{\text{particle}}$$

convective terms

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \underbrace{\frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}}_{} + \frac{\partial f}{\partial t}$$

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial t}dt$$

Given

$\omega = \sqrt[3]{\frac{g}{b}} \rightarrow [b] = Lt$

rel err $= \frac{dg}{g}$

abs err $= dg$

error

$\left[\frac{L}{t}\right]$ $\quad \frac{L}{t^2}$

max in g is 1% — relative error

true $g = g_{nom} \pm 0.01\, g_{nom}$ — absolute error (dg)

max error in $b = 0.5\%$

Asked: max % error in ω

Error rules:

1) when adding or subtracting quantities

[add absolute errors]

2) when multiplying or dividing quantities

[add relative errors]

3) when raising a quantity to a power p, multiply the relative error by $|p|$.

$g \to 1\% = 0.01$ $\qquad b \to 0.5\% = 0.005$

$\frac{g}{b}$ has relative error $0.01 + 0.005$

$= 0.015$

$\left(\frac{g}{b}\right)^{\frac{1}{3}}$ has relative error

$= \left|\frac{1}{3}\right| 0.015 = 0.005 = 0.5\%$

$\left(\frac{g}{b}\right)^{1/3} = w$

Using total differentials

$$d\left(\frac{g}{b}\right) = \frac{b\,dg - g\,db}{b^2} = \frac{g}{b}\left(\frac{dg}{g} - \frac{db}{b}\right)$$

rel err g, rel err b

max err

$$= \frac{g}{b}\left(\left|\frac{dg}{g}\right| + \left|\frac{db}{b}\right|\right)$$

rule 2

$$d\sqrt[3]{\frac{g}{b}} = \frac{1}{3}\left(\frac{g}{b}\right)^{\frac{1}{3}-1} d\frac{g}{b}$$

$$\frac{d\sqrt[3]{g/b}}{\sqrt[3]{g/b}} = \frac{\frac{1}{3}\left(\frac{g}{b}\right)^{\frac{1}{3}-1} d\frac{g}{b}}{\sqrt[3]{\frac{g}{b}}} = \frac{1}{3}\frac{d(g/b)}{g/b}$$

rule 3

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Given: Circular cylinder of radius $r(t)$ and height $h(t)$

at given time $r = 6''$ $h = 8''$

$\dot{r} = 0.2''/s$ $\dot{h} = -0.4''/s$

Asked: at that time $\dot{V}$? $\dot{A}$?

Solution: $V = \pi r^2 h$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$= 2\pi r h \dot{r} + \pi r^2 \dot{h} = 15.08 \text{ in}^3/\text{sec}$$

$$A = 2\pi r^2 + 2\pi r h$$

$$\frac{dA}{dt} = \frac{\partial A}{\partial r}\frac{dr}{dt} + \frac{\partial A}{\partial h}\frac{dh}{dt}$$

$$= (4\pi r + 2\pi h)\dot{r} + (2\pi r)\dot{h} = 10.05 \text{ in}^2/\text{s}.$$