

hi 5060

$$f(x) = \sin^2(x) \quad 2 \frac{x^2}{2!}$$

$$f'(x) = 2 \sin(x) \cos(x)$$

$$f''(x) = 2 - 4 \sin^2(x)$$

$$f'''(x) = -4 f''(x)$$

$$f^{(4)}(x) = -4 f'''(x) = -$$

$$\vdots \quad (-8)(-4) = 32$$

$$f(x) \begin{cases} f(0) = 0 \\ f'(0) = 0 \\ f''(0) = 2 \\ f'''(0) = 0 \\ f^{(4)}(0) = -8 \\ f^{(5)}(0) = 0 \\ f^{(6)}(0) = +32 \end{cases}$$

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$$f(x) = f(a) + f'(a) \frac{x}{1!} + \dots$$

$$= \sum_{n=0}^{\infty} f^{(n)}(a) \frac{x^n}{n!}$$

Here,  $n$  even  $n \geq 2$

$$\sin^2(x) = \sum_{\substack{n=2 \\ n \text{ even}}}^{\infty} 2 (-4)^{\frac{n}{2}-1} \frac{x^n}{n!}$$

$$= 2 \frac{x^2}{2} - 8 \frac{x^4}{4!} + 32 \frac{x^6}{6!} + \dots$$

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~~$$\sum_{k=1}^{\infty} 2 (-4)^{k-1} \frac{x^{2k}}{(2k)!}$$~~ not recommended

for  $n = 2:2:10000$  for  $k = 1:5000$

$$\sin^2(x)$$

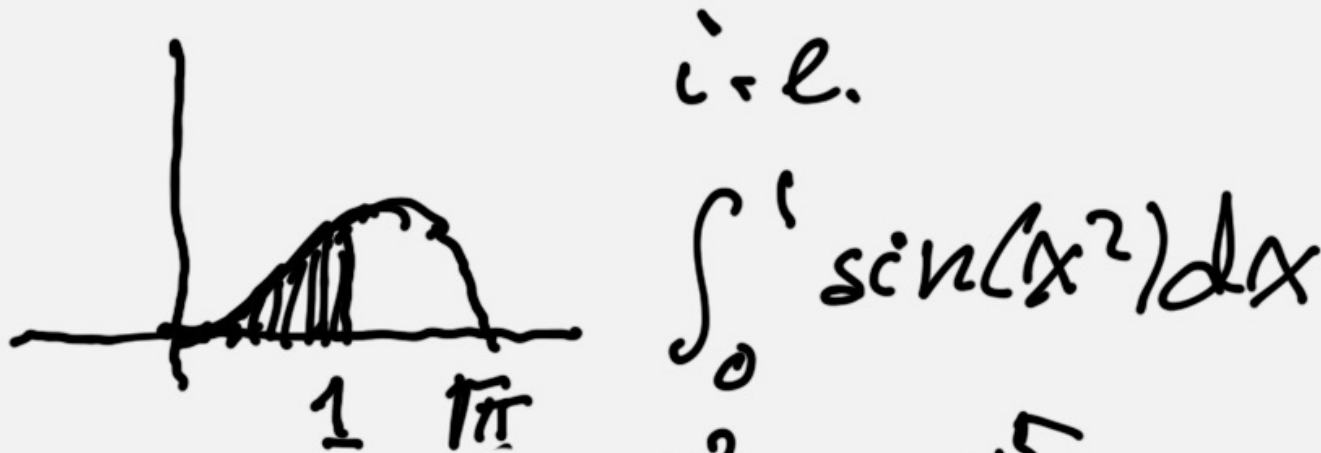
$$\sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$$

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Asked area below  $y = \sin(x^2)$   
 for  $0 \leq x \leq 1$



$$\sin(u) = u - \frac{u^3}{3!} + \frac{u^5}{5!} - \dots$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots$$

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$$\int_0^1 e^{-x} = \left[ \frac{x^3}{3!} - \frac{x^2}{2 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} + \dots \right]_0^1$$

$$\frac{1}{3} - \frac{1}{2 \cdot 3!} + \frac{1}{11 \cdot 5!} + \dots$$

$$0.3103 \pm 0.0008$$

.. ✓

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Limits : l'Hopital:

$$\lim_{x \rightarrow a} \frac{f_1(x)}{f_2(x)} = \frac{f_1(a)}{f_2(a)} \text{ or } \frac{f_1(\pm\infty)}{f_2(\pm\infty)}$$

or  $x = \pm\infty$  unless that value is

$$\frac{\infty}{\infty} \text{ or } \frac{0}{0}$$

then take the ratio of the derivatives

$$\lim_{x \rightarrow} \frac{f_1(x)}{f_2(x)} = \lim_{x \rightarrow} \frac{f_1'(x)}{f_2'(x)}$$

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$$\begin{aligned}
 \lim_{x \rightarrow -\infty} x^2 e^x &= \lim_{x \rightarrow -\infty} \frac{x^2 \rightarrow \infty}{e^{-x} \rightarrow \infty} \\
 &\stackrel{L}{=} \lim_{x \rightarrow -\infty} \frac{2x \rightarrow \infty}{-e^{-x} \rightarrow \infty} \\
 &\stackrel{L}{=} \lim_{x \rightarrow -\infty} \frac{2 \rightarrow 2}{e^{-x} \rightarrow \infty} = 0 = \frac{2}{\infty}
 \end{aligned}$$

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$$\lim_{x \rightarrow 0} (x - \arcsin x) \csc^3 x$$

$$\lim_{x \rightarrow 0} \frac{x - \arcsin x}{\sin^3 x} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{1 - \frac{1}{\sqrt{1-x^2}}}{3 \sin^2 x \cos x} \rightarrow 0$$

$$\lim_{x \rightarrow 0} \frac{-\frac{1}{2} \cdot \frac{1}{(1-x^2)^{3/2}}}{-2 \sin x \cos x} \Rightarrow -\frac{1}{6}$$

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$$x - \left( \cancel{x} + \frac{1}{6} x^3 + \dots \right) = \frac{1}{6}$$

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