EML 5060 **Fall 2009**

Homework Problems

Do not print out this page. Keep checking for changes.
New refers to the 2007 sixth edition, old to the 2003 fifth edition.
Explain all reasoning.

1 9/28

1. New: 6.1: 4, 10 (graph must be neat), 12 (same), 24 (same). Old: 5.1: 4, 10 (graph must be neat), 12 (same), 28 (same).

   ADD

2. New: 6.2: 15. Old: 5.2: 15.
3. New: 6.3: 14. Old: 5.3: 18.
4. New: 6.4: 9, 10, 16. Old: 5.4: 9, 10, 20.
5. New: 6.5: 1, 4, 18. Old: 5.5: 1, 4, 18.
6. New: 7.1: 2, 4. Old: 6.1: 2, 4.
7. New: 7.1: 3, 9, 12. Old: 6.1: 3, 9, 12.

2 10/05

1. New: 6.2.8. Old: 5.2.8. Neatly and clearly graph the plane in 3D space.
2. New: 7.7.13. Old: 6.7.13. Follow the same, completely written out, procedure as used in class for 7.37.
3. New: 7.7.13. Old: 6.7.13. Follow the same, augmented matrix procedure as used in class for 7.37.
4. New: 7.2.6. Old: 6.2.6. ("by 4", not "by row 4") Find Ω as the product of the elementary matrices of the row operations.
5. New: 7.3.6. Old: 6.3,6. Find Ω as the product of the elementary matrices of the row operations. Do it two ways: (a) by first interchanging the two rows; (b) without row exchanges. You should find that Ω_a and Ω_b are not the same. Are the reduced matrices A_{Ra} and A_{Rb} the same, as theorem 7.10 (6.10) claims? Show that if A would have been a nonsingular square matrix, Ω_a and Ω_b would have been the same.

3 10/12

1. New: 7.3.10. Old: 6.3.10. Find Ω by augmenting A by the three by three unit matrix. Use the procedures given in class (i.e. first reduce A to echelon form, and then reduce it farther to row reduced/row canonical form. Verify that indeed $\Omega A = A_R$.
2. New: 7.4.12 Old: 6.4.12 Bases must be cleaned up as much as possible.
3. New: 7.5.6, 7.6.6, 7.6.13, 7.6.14 Old: 6.5.6, 6.6.6, 6.6.21, 6.6.22. Write the general solution of the first problem as a linear combination of basis vectors (as column vectors).
4. New: 7.7.8, 7.7.14 Old: 6.7.8, 6.7.14. Write the general solution as a linear combination of basis vectors (as column vectors) plus a constant vector. Do not reduce to reduced echelon form; reduce to echelon form only and solve it from there.

5. New: 7.8.6 Old: 6.9.6 Use elimination to find the inverse.
6. New: 8.5.6 Old: 7.5.6 *Do NOT use row or column operations!*
7. New: 8.5.6 Old: 7.5.6 *Use row operations ONLY to reduce to upper triangular form!*
8. New: 8.7.8 Old: 7.7.8. Use minors to do so.

4 10/19

1. New: 9.1.4, 9.1.6 Old: 8.1.4, 8.1.6. Find a *complete set* of independent eigenvectors for each eigenvalue. Make sure to write the null space for any multiple eigenvalues. No Gerschgorin. State whether singular and/or defective.
2. New: 9.1.14 Old: 8.1.14 Find a *complete set* of independent eigenvectors for each eigenvalue. Make sure to write the null space for any multiple eigenvalues. No Gerschgorin. Explain whether singular and/or defective or not.
3. New: 9.1.4, 9.1.6 Old: 8.1.4, 8.1.6. Refer to your previous work on these matrices. Check that $E^{-1}AE$ is indeed Λ for the eigenvalues and eigenvectors you found. If not, explain why not.
4. New: 9.2.11 Old: 8.2.13. First, ensure that the book knows what it is talking about by taking the matrix A of 9.1.6/8.1.6,
$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$
and showing that A^2 can be diagonalized. Then check that A can indeed be diagonalized as the book says. If you find that the author knows what he is talking about, prove that the theorem is true for any arbitrary matrix A. If you do not believe that the author has a clue, then prove, for every matrix A, that if A is diagonalizable, A^2 is diagonalizable. Hint: relate the eigenvectors and eigenvalues of A^2 to those of A.
5. New: 9.2.12 Old: 8.2.14. Show first that in the basis of the eigenvectors,
$$A'^k = \Lambda^k = \begin{pmatrix} \lambda_1^k & 0 & 0 & \cdots \\ 0 & \lambda_2^k & 0 & \cdots \\ 0 & 0 & \lambda_3^k & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$
To do so, show first that this is true for $k = 1$. Then show that if it is true for a value k, such as $k = 1$, it is true for the next larger value of k:
$$\begin{pmatrix} \lambda_1^k & 0 & 0 & \cdots \\ 0 & \lambda_2^k & 0 & \cdots \\ 0 & 0 & \lambda_3^k & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 & 0 & \cdots \\ 0 & \lambda_2 & 0 & \cdots \\ 0 & 0 & \lambda_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} \lambda_1^{k+1} & 0 & 0 & \cdots \\ 0 & \lambda_2^{k+1} & 0 & \cdots \\ 0 & 0 & \lambda_3^{k+1} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$
That will imply the desired result for $k = 2, 3, 4, \ldots$ in succession through recursion. Next show that since $A = EA'E^{-1}$
$$A^k = EA'E^{-1}\,EA'E^{-1}\,EA'E^{-1}\,\ldots\,EA'E^{-1} = EA'^kE^{-1}$$
Use the theorem to find a square root of the matrix of 9.2.5/8.2.5, i.e. A^2 is the matrix of question 9.2.5/8.2.5, and you must find A. Indicate $\sqrt{-1} = i$. Note: the eigenvalues and eigenvectors of the 9.2.5/8.2.5 matrix are:
$$\lambda_1 = 0 \quad \vec{e}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \lambda_2 = 5 \quad \vec{e}_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} \qquad \lambda_3 = -2 \quad \vec{e}_3 = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$$

6. New: 9.1.4 Old: 8.1.4. Refer to your previous work on the matrix. For the given matrix solve the system $\dot{\vec{x}} = A\vec{x}$ using the same method of diagonalization as used in class. Accurately draw a comprehensive set of typical solution curves in the x_1, x_2 plane.

5 10/26

1. New: 9.3.6 Old: 8.3.6. Orthogonal means orthonormal. Determine the inverse of the orthogonal matrix using the class procedure only.

2. Find an orthonormal matrix that diagonalizes the matrix

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

   Hint: one eigenvalue is 1.

3. New: 9.4.16 Old: 8.4.16 Also accurately draw the conic in the x_1, x_2-plane. List the angles of the various axes and asymptotes.

4. New: 9.4.18 Old: 8.4.18 Also accurately draw the conic in the x_1, x_2-plane. List the angles of the various axes and asymptotes. Write out explicitly the relations that compute the old coordinates from the new ones and vice/versa.

5. Describe the surface $\vec{x}^T A\vec{x} = 4$ for the matrix of question 2. What is its shape? How is it oriented?

6 11/04

1. New: 1.1.18 Old: 1.1.22 Derive the qualitative properties (symmetries, asymptotes, maxima, minima, ranges, cusps, inflection points) of the curve from the direction field. *Do not solve the ODE; graphics only.* No cheating!

2. New: 1.2.14 Old: 1.2.14 Also solve it when $y(1) = 0$. Neatly sketch both solutions in the x, y-plane.

3. New: 1.3.4 Old: 1.3.4. Use the class procedure, variation of parameter.

4. New: 2.4.2 Old: 2.4.2

5. New: 2.6.4 Old: 2.6.4. Must use variation of parameters.

6. New: 2.6.16 Old: 2.6.16. Must use undetermined coefficients.

7. New: 2.6.32 Old: 2.6.50. Must use undetermined coefficients.

7 11/13

Must use Laplace transform in all questions. You wish.

1. New: 3.1.6 Old: 3.1.6

2. New: 3.2.2 Old: 3.2.2

3. New: 3.2.8 Old: 3.2.8

4. New: 3.3.30 Old: 3.3.34

5. New: 3.3.6 Old: 3.3.6

8 11/20

1. New: 3.4.14 Old: 3.4.14
2. New: 3.5.2 Old: 3.5.2 Graph neatly.
3. New: 3.5.8 Old: 3.5.8
4. New: 3.6.16 Old: 3.6.16. "Solve" means here find $\hat{y}_1$ and $\hat{y}_2$, not y_1 and y_2.
5. New: 10.2.10 Old: 9.2.12. Write first the general solution to the system, regardless of initial conditions in terms of the fundamental matrix. Then plug in the initial conditions and clean up.
6. New: 10.2.16 Old: 9.2.18

9 11/30

1. New: 10.2.28 Old: 9.2.36. Use the stated method.
2. New: 10.2.44 Old: 9.2.60. Solve by finding $e^{At}\vec{v}$-type solutions for suitable vectors $\vec{v}$. Identify matrix e^{At}.
3. New: 10.3.4 Old: 9.3.4
4. New: 10.3.8 Old: 9.3.8

10 12/04

1. New: 11.3.8 Old: 10.3.8 Classify the critical point. State the type of stability. Draw eigenvectors, or their real and imaginary parts, and solution curves accurately. Use a ruler and measure it. Neatly draw at least two solution curves in every distinguishable region. Put direction arrows on all the curves. Make sure the correct slopes can clearly be distinguished on the solution curves at large positive and negative times.
2. New: 11.3.10 Old: 10.3.10 Classify the critical point. State the type of stability. Draw eigenvectors, or their real and imaginary parts, and solution curves accurately. Use a ruler and measure it. Neatly draw at least two solution curves in every distinguishable region. Put direction arrows on all the curves. Make sure the correct slopes can clearly be distinguished on the solution curves at large positive and negative times.
3. New: 11.5.2 Old: 10.5.2
   (a) Find the critical points. One critical point is easy. More critical points can be found numerically. In particular their y-values are ± 1.1107 and ± 1.6074.
   (b) Find the matrix of derivatives of vector $\vec{F}$ at each critical point.
   (c) Use it to analyze each critical point. List type of point and its stability. Sketch the solution lines in the immediate vicinity of each critical point.
4. Draw comprehensive solution curves based on the critical points and a grid of local slopes.
5. Compare the picture you got in the previous question quantitatively with the positions of the critical points and the directions of the eigenvectors that you got using critical point analysis. State whether critical point analysis *must* give the right solution near the point, and whether it does.

For the second last question, you will want to use some computer program to plot or at least print out slopes at say 30 times 30, or 900 points. If you are willing to log onto unix and run a fortran program, a link is here.[1]

A better solution may be to use a direction field program from the web. The one that seems nicest to me is this one.[2] Also found here.[3]

See here for Matlab software.[4] (You will need to convert to an ODE by taking the ratio of the equations, and then the software might crash when it divides by zero if it hits a critical point.)

Another to try is here.[5]

The Windows screen-grabber I use is called Printkey. I am sure there are others.

[1] http://www.eng.fsu.edu/~dommelen/courses/aim/slopes

[2] http://www.math.uu.nl/people/beukers/phase/newphase.html

[3] http://www.math.psu.edu/melvin/phase/newphase.html

[4] http://math.rice.edu/~dfield/index.html

[5] http://people.scs.fsu.edu/~burkardt/m_src/direction_arrows_grid/direction_arrows_grid.html