EML 5060 **Fall 2007**

Homework Problems

Do not print out this page. Keep checking for changes.
New refers to the 2007 sixth edition, old to the 2003 fifth edition.

1 10/1

1. New: 6.1: 4, 10, 12, 24. Old: 5.1: 4, 10,12, 28
2. New: 6.2: 15. Old: 5.2: 15.
3. New: 6.3: 14. Old: 5.3: 18.
4. New: 6.4: 9, 10, 16. Old: 5.4: 9, 10, 20.
5. New: 6.5: 1, 2, 4, 18. Old: 5.5: 1, 2, 4, 18.
6. New: 7.1: 2, 4. Old: 6.1: 2, 4.
7. New: 7.1: 3, 9, 12. Old: 6.1: 3, 9, 12.

2 10/08

1. New: 7.7.13. Old: 6.7.13 (the same two ways as in class)
2. New: 7.2.6. Old: 6.2.6. ("by 4", not "by row 4")
3. New: Prove theorem 7.10. Old: Prove theorem 6.10.
4. New: 7.3.10. Old: 6.3.10

3 10/15

1. New: 7.4.12 Old: 6.4.12 (Bases must be cleaned up.)
2. New: 7.5.6, 7.6.6, 7.6.13, 7.6.14 Old: 6.5.6, 6.6.6, 6.6.21, 6.6.22
3. New: 7.7.8, 7.7.14 Old: 6.7.8, 6.7.14
4. New: 7.8.6 Old: 6.9.6 (Use elimination.)
5. New: 8.5.6 Old: 7.5.6 *No row or column operations!*
6. New: 8.5.6 Old: 7.5.6 *Row operations only, no minors!*

4 10/22

1. New: 8.7.8 Old: 7.7.8. Using minors, unless you used minors in 7.8.6/6.9.6, then using elimination.
2. New: 9.1.4, 9.1.6 Old: 8.1.4, 8.1.6. Find a *complete set* of eigenvectors. No Gerschgorin. State whether singular and/or defective.
3. New: 9.1.14 Old: 8.1.14 Find a *complete set* of eigenvectors. No Gerschgorin. State whether singular and/or defective.
4. New: 9.1.4, 9.1.6 Old: 8.1.4, 8.1.6. Redux. Check that $E^{-1}AE$ is indeed Λ for the eigenvalues and eigenvectors you found. If not, explain why not.

5. New: 9.2.5 Old: 8.2.5 Verify by multiplication that $E^{-1}AE$ is indeed Λ.

6. New: 9.2.11 Old: 8.2.13 This is another of these questions the students can easily answer. First, for the matrix A of 9.1.6/8.1.6, find A^2 and comment on the problem. Then prove, for any matrix A, that if A is diagonalizable, A^2 is diagonalizable. Hint: find eigenvectors and eigenvalues of A^2 in terms of those of A.

7. New: 9.2.12 Old: 8.2.14. Note that if $E^{-1}AE = \Lambda$, then $A = E\Lambda E^{-1}$. Then show that if the theorem is true for any value k, including $k = 1$, it is true for the next larger value of k, implying the theorem by recursion. Use the theorem to find a square root of the matrix of 9.2.5/8.2.5, i.e. a matrix A whose square is the matrix of question 9.2.5/8.2.5. Indicate $\sqrt{-1} = i$.

5 10/29

1. New: 9.3.6 Old: 8.3.6

2. New: 9.4.18 Old: 8.4.18 Also accurately draw the conic in the x_1, x_2-plane.

3. New: 9.2.5 Old: 8.2.5. Write the transformation matrix from the original Cartesian coordinate system to the coordinate system that uses the eigenvectors as basis. Give formulae that compute the Cartesian coordinates (multiplying the Cartesian basis vectors) in terms of the "new" coordinates multiplying the eigenvectors. Also give the formulae that work vice-versa.

4. New: 9.1.4 Old: 8.1.4. Redux. Solve the system $\dot{\vec{x}} = A\vec{x}$ and draw typical solution curves in the x_1, x_2 plane.

6 11/07

1. New: 1.1.18 Old: 1.1.22 Derive the qualitative properties (symmetries, asymptotes, maxima, minima, ranges, cusps, inflection points) of the curve from the direction field. Do not solve the ODE to do so.

2. New: 1.2.14 Old: 1.2.14 Also solve it when $y(1) = 0$. Neatly sketch both solutions.

3. New: 1.3.4 Old: 1.3.4

4. New: 2.4.14 Old: 2.4.16

5. New: 2.4.2 Old: 2.4.2

6. New: 2.6.4 Old: 2.6.4

7. New: 2.6.16 Old: 2.6.16

8. New: 2.6.32 Old: 2.6.50

7 11/16

1. New: 3.1.6, 3.1.18 Old: 3.1.6, 3.1.18

2. New: 3.2.2 Old: 3.2.2

3. New: 3.2.8 Old: 3.2.8

4. New: 3.3.30 Old: 3.3.34

5. New: 3.3.6 Old: 3.3.6

8 11/26

1. New: 3.4.6 Old: 3.4.6 Using convolution; work out completely.
2. New: 3.4.14 Old: 3.4.14
3. New: 3.5.2 Old: 3.5.2 Graph neatly.
4. New: 3.5.8 Old: 3.5.8
5. New: 3.6.16 Old: 3.6.16. "Solve" means here find $\hat{y}_1$ and $\hat{y}_2$. The controls students will comment of the effectiveness of the device as a vibration absorber.
6. New: 10.1.2 Old: 9.1.2. Question (a) means verify that the solution is no good, but ignore that in (b)-(d).
7. New: 10.2.4, 10.2.10 Old: 9.2.4, 9.2.12. Use the system given in 10.2.10/9.2.12 in both questions.

9 12/03

1. New: 10.2.16 Old: 9.2.18
2. New: 10.2.28 Old: 9.2.36
3. New: 10.2.44 Old: 9.2.60
4. New: 10.3.8 Old: 9.3.8

10 12/07

1. New: 11.3.2 Old: 10.3.2 Classify the critical point. State the type of stability. Draw eigenvectors, or their real and imaginary parts, and solution curves accurately. Use a ruler and measure it. Neatly draw at least one solution curve in every distinguishable region. Put direction arrows on all the curves. Make sure the correct slopes can clearly be distinguished on the solution curves at large positive and negative times.
2. New: 11.3.6 Old: 10.3.6 Same requirements as the first question.
3. New: 11.3.8 Old: 10.3.8 Same requirements as the first question.
4. New: 11.3.10 Old: 10.3.10 Same requirements as the first question.
5. New: 11.5.2 Old: 10.5.2 One critical point is easy. More critical points can be found numerically. In particular their y-values are ± 1.1107 and ± 1.6074.
6. Draw comprehensive solution curves based on the critical points and a grid of local slopes.
7. Compare the picture you got in the previous question quantitatively with the positions of the critical points and the directions of the eigenvectors that you got using critical point analysis.

For the second last question, you will want to use some computer program to plot or at least print out slopes at say 30 times 30, or 900 points. If you are willing to log onto unix and run a fortran program, a link is here.[1]

A better solution may be to use a direction field program from the web. The one that seems nicest to me is this one.[2] Also found here.[3]

[1] http://www.eng.fsu.edu/~dommelen/courses/aim/slopes
[2] http://www.math.uu.nl/people/beukers/phase/newphase.html
[3] http://www.math.psu.edu/melvin/phase/newphase.html

See here for Matlab software.[4] (You will need to convert to an ODE by taking the ratio of the equations, and then the software might crash when it divides by zero if it hits a critical point.)

Another to try is here.[5]

The Windows screen-grabber I use is called Printkey. I am sure there are others.

[4] `http://math.rice.edu/~dfield/index.html`

[5] `http://people.scs.fsu.edu/~burkardt/m_src/direction_arrows_grid/direction_arrows_grid.html`