Tentative list.

EML 5060 **Homework Set 3** **Fall 2006**

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below	1		Predator-prey
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below	3		Van der Pol oscillator

1. The predator-prey problem is:

$$\dot{x}_1 = ax_1 - bx_1x_2 \qquad \dot{x}_2 = cx_1x_2 - dx_2$$

   where a, b, c, and d are positive constants. The product x_1x_2 is a measure of how frequently predators and preys meet; note that such meetings decrease the number of preys, but benefit the predators.

   Find the two critical points of this system and classify them according to type. What can you say about the topology of the linearized solution near the two points? And what about the topology of the full nonlinear solution?

2. Take the ratio of the two predator-prey equations to eliminate time, and solve the resulting first order ODE. Determine whether you can now be more specific about the nonlinear topology of the solution

curves. Also note that the x_1 and and x_2 axes are solution curves; examine the direction of their arrows. Now sketch the complete set of solution curves.

3. Classify the topology of the critical points of the Van der Pol equation

$$\ddot{\theta} + c(\theta^2 - 1)\dot{\theta} + \theta = 0$$

*: Recommended question. Not required if you know you can do it.

#: Make a graph. For problems with more than one unknown parameter, draw the independent solutions. For systems, draw the solution/ all solutions in the phase plane.

[1]: Simplify the transformed solution using partial fractions.

[2]: Solution appears to be wrong.

[3]: Solve as a system, but do *not* use $e^A t$. Use eigenvectors.

[4]: Sketch the solution curves using the actual eigenvectors.

[5]: Ignore the given initial conditions in sketching the solution curves.