

REEF Subsonic WT Articulating Robotic Arm



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Abstract

The goal of the REEF WT Articulating Robotic Arm project is to create a robotic arm capable of mounting, pitching, and yawing a specimen during operation of the wind tunnel. During operation, the mounting mechanism must keep the specimen in the center of the 42in² test section. The previous arm for the wind tunnel was relocated to another research facility, and a new one is required to carry out further testing of specimens. A sting mount will be utilized in order to minimize flow disruption around the specimen. The joints and base of the arm will be moved using stepper motors. Design constraints, performance specifications, and a schedule have been created. A Gantt chart shows tentative schedule for the remainder of the semester. The selected design idea will now be drafted with respect to the constraints and specifications. The next deliverable will offer a detailed final design with intent to produce a prototype during the spring semester.

1 Introduction

The objective of this project is to create a mechanism to mount a specimen in the center of the wind tunnel test area. This mechanism must be able to adjust the pitch and yaw of the specimen while the wind tunnel is operational. A servo control unit will be provided to be programmed with the purchased stepper motors and the user interface. These stepper motors will be the source of movement for the mechanism. The wind tunnel has a maximum speed of 22 m/s, or approximately 50 mph, with a 42in² test section. A sting mount will be used to hold the specimen in place. Multiple mechanisms of this type exist; the background portion of this paper analyzes a few different mounting types used for research in large wind tunnels.

Numerous problems remain to be solved in this project. First, a design must be created in order to best adjust orientation of the specimen while keeping it located in the center of the test section. Changes of the model location within the flow could lead to undesirable results. The team will have to decide on an angle of attack as well as design the mechanism to move the specimen in pitch and yaw. Second, forces from the wind tunnel must be analyzed in order to build a structure that can withstand maximum speeds. A high factor of safety will be used for this design portion so that the integrity of the mounting mechanism is ensured. Third, a force reducing mechanism such as a gearbox or chain drive must be designed in order to move the mounting mechanism during wind tunnel operation. This will also incorporate the force analysis on the tunnel. The final problem is a result of financial limitations. The components that are intended to be used in the design must fall within the budgeted of \$2,000.

2 Project Definition

2.1 Background research

Wind tunnels have proven to be a cost effective means to test an aerodynamic design in a controlled environment. Small scale aircraft models will have the same drag, lift, and side force coefficients as full scale aircraft in flight. In order to properly test an object in a wind tunnel, a device must be constructed to hold the model in place and measure the forces acting on it. Depending on the desired data, model size, and wind tunnel test section, the mount could be very robust, or be very discreet to reduce impact on the acquired data. There are several types of mounts that have been developed for wind tunnel testing. Four commonly used mounts are single strut, two strut, three strut, and sting mounts² as shown in figure 1.

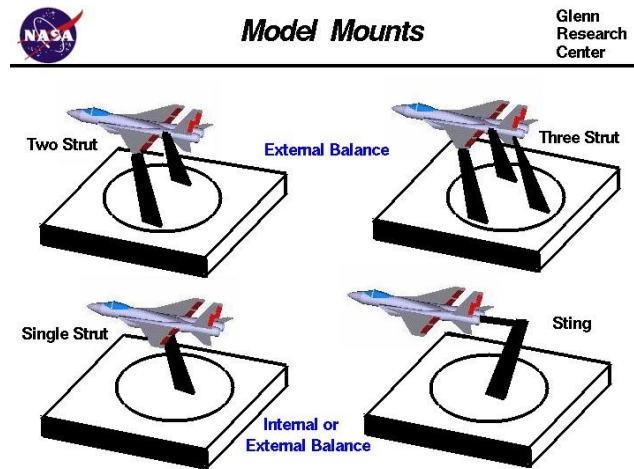


Figure 1: Model Mounts²

Per suggestion of our sponsor, the mount we will utilize is a sting mount. The benefit of the sting mount is there is little areodynamic interference until the flow reaches the wake. This means the lift and side forces will be unaffected, however, the drag force will be slightly impacted by the mount geometry itself. Sting mounts also provide an easy method to run wires or tubes through the mount and to the control room.

Sting mounts are very versatile and have the benefit of providing internal or external balance testing. With internal testing, strain gages are placed within the sting assembly inside the aircraft model. These strain gages will measure the forces

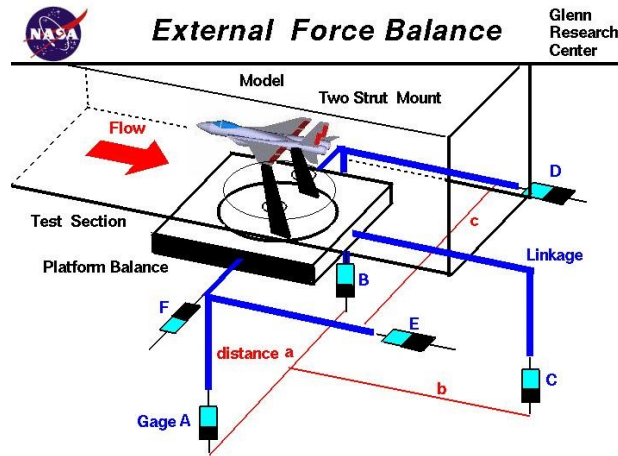


Figure 2: External Force Balance⁴

and moments acting on the model. The lift, drag, and pitch can be determined. However, the side forces (roll or yaw) can't be determined with internal balance testing³. In order to measure the side forces, the mount must be able to perform an external balance. The external balance incorporates multiple strain gages within the base of the model itself. In figure 2, boxes A through F represent the different strain gages within the base of the mount. These gages measure six different components, lift, drag, and side forces, as well as pitch, yaw, and roll moments⁴.

2.2 Need Statement

The sponsor for team 12, the REEF Subsonic WT Articulating Robotic Arm project, is the Air Force Research Lab. Mike Systma is the Air Force Research Lab representative for this project. The facility has a subsonic wind tunnel with a test section of 42 in². The wind tunnel reaches a maximum speed of 22 meters/second. The existing robotic arm mount was removed and placed in a different wind tunnel. A new robotic arm must be designed in order to mount test specimens. The test specimens must be able to adjust in pitch and yaw within the center of the wind tunnel test section.

Need Statement: There is no mounting mechanism in the wind tunnel to hold the specimen.

2.3 Goal Statement & Objectives

Goal Statement: Design a mounting mechanism in order to mount and adjust test specimen to desired orientations during wind tunnel operation.

Multiple objectives have been set forth to be achieved in the design of the mounting mechanism. While a test specimen is being held in the active flow of the wind tunnel the mechanism must be able to manipulate the orientation of the specimen. The angle of attack (pitch) of any specimen must be able to be adjusted 30° above or below a sitting position of completely level. The yaw (side slip) of the specimen must also be able to be adjusted 20° to the left or right of an initial position of being directly aligned with the flow. While the orientation of the specimen is being shifted the location of the model; the middle of the test section; must remain the same so that consistent results may be achieved. Once the specimen has been shifted to a desired orientation it is pivotal that the model remain still and refrain from moving or swaying. For this

to be achieved the mechanism should be designed to withstand the maximum forces that can be produced by the wind tunnel at its highest velocity, 22 m/s.

Objectives:

- Adjust pitch (angle of attack) of specimen $\pm 30^\circ$
- Adjust yaw (side slip) of specimen $\pm 20^\circ$
- Keep specimen in center of test section
- Withstand maximum wind speeds of the tunnel
- Hold specimen still

2.4 Constraints

There are multiple constraints that need to be acknowledged and adhered to for the production of a robotic arm for use in a subsonic wind tunnel. The arm is required to alter the pitch and either the roll or yaw of a given model as it is studied in a wind tunnel, based on parameters inputted by researchers that will be carried out by the mechanism's stepper motors. There will be two stepper motors to adjust the pitch and yaw. The model must maintain a position in the center of the flow while the pitch and yaw are changed. The power source for the robot would come from a standard wall socket and converted into a DC current via a power supply. This power demand will depend on the selected stepper motors.

The first main constraint is the budget that has been allotted for the project, a total of \$2,000, for the procurement of materials and construction of the arm. The major expenditures come from the purchase of stepper motors and encoders along with raw materials that will need to be machined. The most expensive part, the servo controller unit, will be provided by the sponsor. A preemptive break down of the budget is shown in Table 1.

Since there is a potential for deformation and even damage to the structure due to the forces produced by the wind tunnel, a high factor of safety is needed. The mounting mechanism and arm must be able to withstand the forces produced by the wind tunnel blowing directly onto both, as well as not tip over due to the previously mentioned forces and lift generated due to the model. This will be helped by the mounting mechanism base that will be produced by the sponsor, constructed mainly out of 80/20; a strong and rigid aluminum building material. The vertical

position of the model held by the arm needs to be placed in the center of a 42"x42" square inlet; the centroid of the opening being approximately 84" in height.

Table 1: Estimated Budget

Item	Estimated Costs
Stepper Motor/Encoders	\$400
Raw Materials	\$600-700
Shop Time/Fabrication	\$400
Total	\$1500

3 Design and Analysis

3.1 Functional Analysis

The process by which the mounting mechanism will operate can be broken down into two distinct sections; the user interface to the controller and the controller to the stepper motors which will in turn operate the stepper motors.

The mounting mechanism will be actively controlled by the user, shifting and adjusting as they require. Therefore a user interface will be required so that commands may be feed to the servo controller. The most likely UI will consist of a command prompt in which the manipulations to the models alignment can be typed; these commands will be feed to the controller.

Once the commands have been passed from the servo controller to the stepper motors, they will initiate their operation. The two motors will be controlled independently, one changing the angle of attack and the other adjusting the yaw by controlling rotation of a turn table provided by the sponsor. The stepper motors will be connected to a force reducing system; increasing the torque produced by the motors; allowing them to easily adjust the pitch and yaw through their full range of desired motion.

Table 2: Functional Analysis

Equipment	Function
Controller	Used to pass command from user interface to respective stepper motors
Stepper Motor at Base	Motor used to adjust the angle of attack (pitch) of the chosen mount design
Stepper Motor/Turn Table	A turn table (provided by sponsor) that has already been integrated with a stepper motor

3.2 Design Concepts

Our first design concept was a simple sting mount with a recessed portion to allow the base to rotate about the centroid of our model.

This design is extremely simple to produce as some bent metal tube, or some separate straight portions screwed together would produce a satisfactory product. It is a very useful design for yaw calculations, as the rotation about the model's centroid would prevent any translation in the flow field, ensuring accurate data. However, this design is only advantageous for very low angles of attack. Once the model is adjusted to some angle of attack, the model will be moved in the flow field as well as no longer rotate about its centroid. This would skew the data to the point that it is no longer useful. With these considerations and the input from our sponsor this design would not meet the necessary requirements i.e., it would not keep the model in the same flow field as angle of attack and yaw are adjusted, and cannot be used.

In order to maintain the model location within the flow field, a design incorporating a circular arc was developed. The arc design allows for the angle of attack, or pitch, to be changed



Figure 3: Design 1

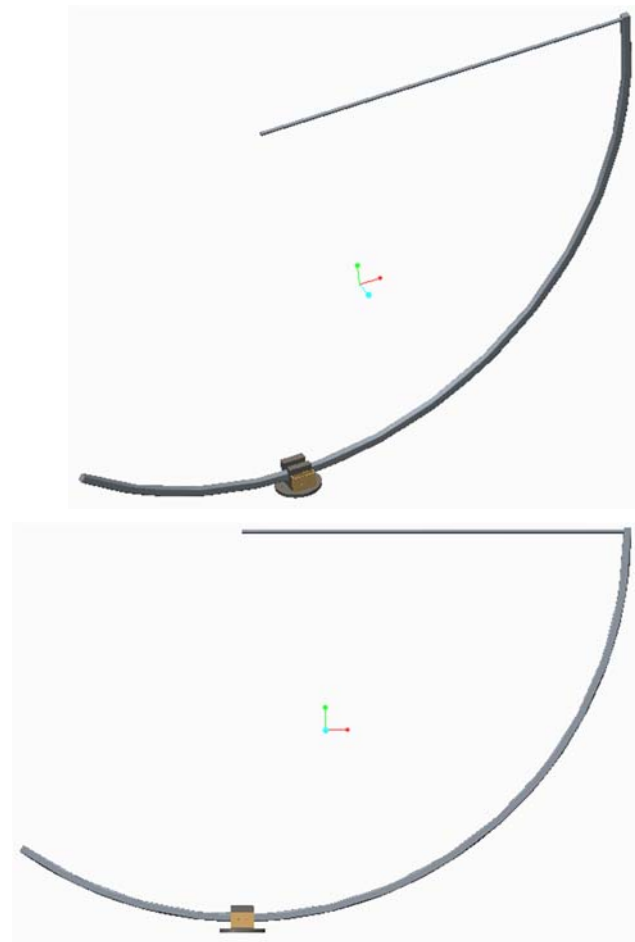


Figure 4: Design 2

The arc design allows for the angle of attack, or pitch, to be changed

within the flow field without translating the model in any direction. By placing the pitch center of rotation in line with the same axis as the yaw rotation, the model can be adjusted during operating conditions without data corruption. The arc design however is more complex to manufacture. The radius of the arc will have to be three to six feet, in order to maintain an adequate distance from the incoming flow to not interrupt flow patterns. This will be very difficult to find off the shelf, and will probably have to be custom made. The arc will be mounted on a turn table with a built in servo motor. This will turn the arc on the horizontal plane, or adjust the yaw. The pitch will be adjusted by a stepper motor and a gear train, most likely utilizing a worm gear against the arc. The worm gear will allow a fine degree of control as well as, being non-back drivable, maintain its position under a load.

Similar to design two, design three utilizes an arc to maintain the model location during dynamic testing conditions. This design was developed to help mitigate the amount of material in front of the model, and maintain designed flow conditions upstream. By placing the arc mounting mechanism at the rear of the arc, the amount of material in the arc is reduced. This reduction in the amount of arc used could result in significant cost savings if it must be custom manufactured. The arc mounting mechanism would have to be moved in both the horizontal plane and vertical plane by the at least the radius of the arc itself. By translating the mounting mechanism by three to six feet, huge moments are created. This would require more materials, resulting in larger motors, higher costs, and overall lower efficiency.



Figure 5: Design 3

3.3 Evaluation of Designs

Designs were evaluated on strength, cost, efficiency, and complexity. In order to evaluate strength, structural analysis had to be completed. Cost was evaluated based on the amount of material that was required and efficiency was measured based on how well the design would accomplish the design task. Finally, complexity was a measure of difficulty from an analytical standpoint. A structure that required more complex calculations and more assumptions would have been scored a lower value in the complexity column.

FEA was completed at the base of each design because this is where the largest moments would occur. The design was analyzed in its most extreme conditions in order to capture the largest expected stresses that could occur on the structure. In both figure xx and figure xxb, there are compressive weight forces and forces from the tunnel. The $Force_{weight}$ would account for the mass for the structure along with the mass of the specimen being tested. $Force_{tunnel}$ accounts for the expected lift from the tunnel. $Torsion_{tunnel}$ accounts for rotational shear stress being induced from the specimen at a large yaw angle. Finally, $shear_{tunnel}$ accounts for the normal shear stress induced from the tunnel wind onto the structure. These equations are shown below in equations 1-3.

$$F_{x_drag} := \frac{1}{2} \cdot \rho_{air} \cdot V_{max}^2 \cdot A_{max} \cdot \sin(\alpha_{max}) \quad Eq.1$$

$$Moment_{max} := F_{x_drag} \cdot radius_{arc} \quad Eq.2$$

$$Torque_{max} := L_{sting} \cdot \cos(\beta_{max}) \cdot F_{x_drag} \quad Eq.3$$

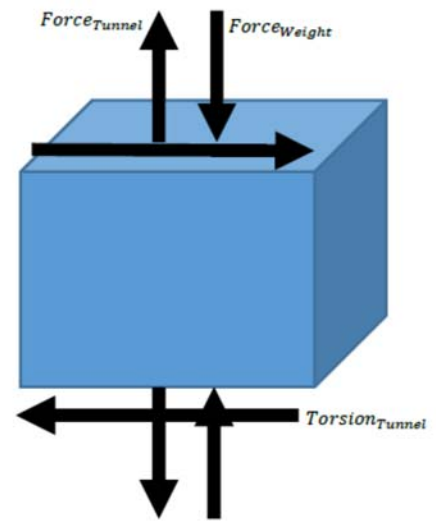


Figure 6: FEA front face mount

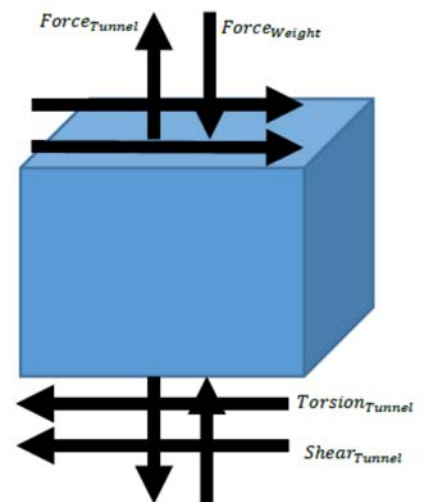


Figure 7: FEA side face of mount

The values for our selected design can be seen in the appendix in the Mathcad calculation section. Using these forces, one could figure out the principle stresses and compare them to the accepted values of aluminum in compression, shear, and tension. The principle shear and normal stress equations are shown below.

$$\sigma_x = \sum \frac{F_x}{d_{arc}} \quad \text{Eq.4}$$

$$\sigma_y = \sum \frac{F_y}{d_{arc}} \quad \text{Eq.5}$$

$$\sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \mp \sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + \tau_{xy}^2} \quad \text{Eq.6}$$

$$\tau_{max} = \sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + \tau_{xy}^2} \quad \text{Eq.7}$$

The decision matrix shown in table 3 shows how the designs were scored based on the evaluation criteria. In the following sections, scoring of these criteria will be discussed along with the reasoning for the chosen optimum design.

Table 3: Decision Matrix

Criteria	Strength	Cost	Efficiency	Complexity	Total
Weight	9	6	9	6	30
Design #1	50	75	75	80	68.5
Design #2	80	65	90	75	79
Design #3	60	50	80	50	62

3.3.1 Criteria

The criteria we chose to judge our potential designs upon follows those shown in the decision matrix; strength, cost, efficiency and complexity. The strength of the design was determined through use of the equations shown above and based on the geometric properties of the models. For example, design #3 shows a high potential for bending at most of the weight is being supported by a single rod that extends away from the center of the design; resulting in its low score. The strength of the design was given a high weighting as the sponsor has requested a

factor of safety on the design of 5, meaning that under no circumstances should the mount ever come close to failing.

The cost portion of the score was mainly determined by the section of the design that will shift the pitch; as the yaw of all three designs will be adjusted in the same manner. Designs #2 and #3 would require a very precise arc to be purchased or fabricated which could potentially cost double to triple what the sting mount for design #1 would be comprised of. The complexity of the design and the cost of the design are interrelated and share the same general trend in the decision matrix; design #3 scoring the lowest and design #1 scoring the highest.

The efficiency component of the decision matrix took multiple factors into account to produce its score. The amount of moving parts required for the design was one factor in which all three design scored the same as the all have few moving parts. Another consideration was the amount of flow interruption in comparison to stability. The mount must refrain from interrupting the flow before it reaches the test model and must maintain a stable unmoving position. Design #2 scored the highest on this portion as it is completely out of the flow, as well as being mounted directly to the turn table will be very stable.

3.3.2 Selection of Optimum Design

Design 2 was the only design that stress analysis was used on. The other criteria halted analysis on the other designs. Design 1 was out of budget because of the required additional motors, and design 3 required further complex analysis. Also, design 3 had a large arc located inside the flow and would not be ideal in the subsonic flow.

The cost was analyzed based on the amounts of material and additional equipment required. Designs 1 and 2 have more material due to the moment arm that extends to attach behind the object. Also, design 1 requires more motors in order to keep the objects center of mass stationary. This would require significant more costs due to the stepper motor and encoder combination. For this reason, design 2 scored the best. It would only require one additional stepper motor and would require material just for an arc.

Efficiency grading was developed based on how easily the mechanism will move. In design 2, one movement covers both translation and centering. It also has just the sting mount in the flow. Design 3 has similar efficiency but has a large arc in the flow. Since the tunnel is subsonic, this would change results due to the fluid being able to adjust to the upcoming boundary. Design

1 had the lowest efficiency because it would require 4 movements to accomplish the same task as designs 2 and 3 accomplish with 2 movements. This also ties into the complexity grading. Although design 1 is extremely simple, it has complexity in terms of keeping the mount centered in the flow. Design 2 has complexity about its arc, but generally does not interrupt the flow patterns. Finally, design 3 has a relatively large object in the flow. This would be hard to analyze and would require many assumptions. It would also require research beyond what is known. For this reason, it scored low.

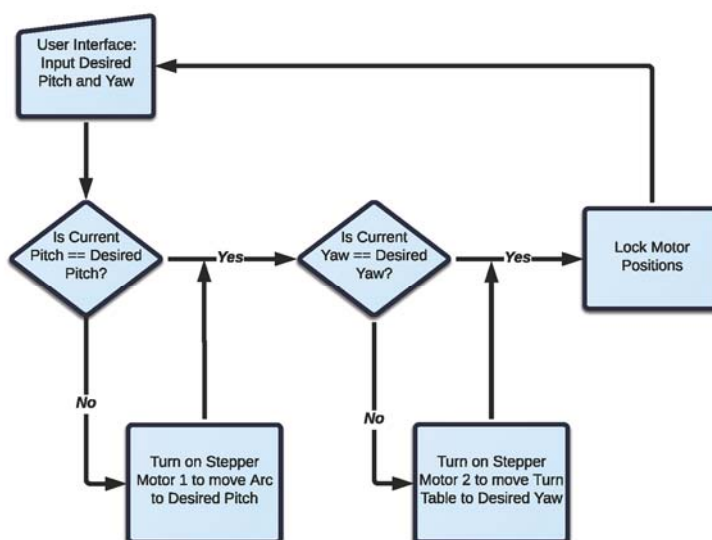
Overall, design 2 was the final selection. This was because it was the most optimized case of strength, cost, efficiency, and complexity. Further analysis will have to be completed in order to decide the best way to move the arc. The strength, cost, efficiency, and complexity of these next designs will be evaluated in the next midterm. Motor selection and analysis will also be completed in this time period.

4 Methodology

Now that the team has visited the site, the next step will be to make informed decisions regarding the structure. The dimensions of the structure will be chosen, as well as a primary material. This will enable the team to perform a more comprehensive FEA. A high factor of safety will still be used for all calculations. With these calculations complete, and checked against constraints, the team can choose adequate stepper motors for the design. Based on these stepper motor requirements, a basic circuit analysis will be performed to determine the power needs of the mechanism. With this circuit analysis, the team can begin the process of designing the circuits that will power the mechanism and the programming that will run the system. Drawings of all parts and circuits will be formed for review by the advisor and sponsor.

Once the designs are approved, an official budget and bill of materials will be formulated. While the design was formed with budget in mind, this portion will gauge actual prices. If the design exceeds the budget provided, it will be re-evaluated for aspects that can be adjusted or redesigned to lower the cost. If changes are made, the analysis process must be completed again. If the design can't be changed, further funding will be requested from the sponsor.

Figure 6 displays the basic processing and functions that the design will incorporate. The user will input desired values for the pitch and yaw of the test specimen. If the desired angles do



not match the current position of the specimen, the motors will turn on to move the specimen. It is important to note that only one angle (pitch or yaw) will change at a time. Also, a new angle cannot be entered until the motors have completed their current movements. Once the specimen reaches the desired position, the user will be able to input new angle values.

Figure 8: Processing and functions

4.1 Schedule

To help keep track of this project and the many design decisions that must be made in order to proceed, the team has formulated a Gantt chart displayed in Figure 3 accompanied by a detailed breakdown. This will enable the team to keep track of progress and make sure that we complete milestones in a timely manner, so as to best prepare us for fabrication in the spring.

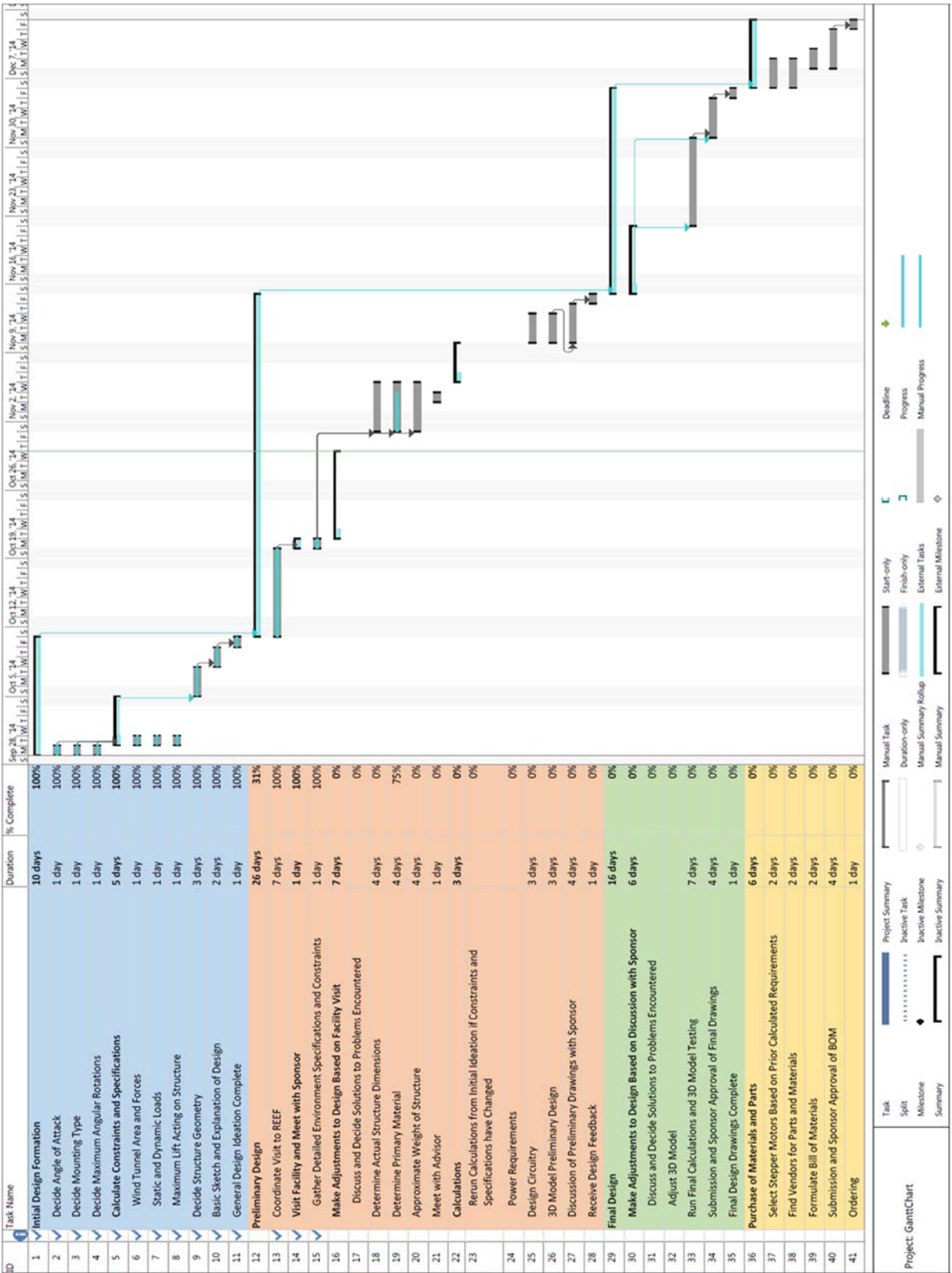


Figure 9: Gantt Chart

4.2 Resource Allocation

Design ideation is a team effort. All major design decisions are discussed by the team and each member contributes ideas to accomplish specifications within the project constraints while also being aware of possible problems that may occur with each idea or change. Table X???? shows the upcoming tasks that require completion for the project to move forward. Each team member has been assigned tasks based on their areas of expertise and has estimated the hours they require to complete those tasks.

Table 4: Resource allocation

Task	Responsible Party	Estimated Hours
Research Materials	Jacob Kraft	1
Calculate FEA on Structure	Jacob Kraft	3
Calculate Power Requirements	Jacob Kraft	1.5
3D model (proE)	Justin Broomall	5
Stress Analysis (Adams)	Justin Broomall	2
Circuit Design Calculations	Caitlan Scheanwald	2.5
Circuit Drawings	Caitlan Scheanwald	2
Research Stepper Motors	Caitlan Scheanwald	2
Research Servo Unit	Caitlan Scheanwald	1
Design Gear Train/System	Andrew Baldwin	4
Calculate Necessary Gear Ratios	Andrew Baldwin	3

Primarily, this group intends to function as a team. While we have assigned specific responsibilities to each member, we also recognize that it is beneficial to work together, especially when certain portions of the design process may be heavier on one team member than on another. Team 12 will work together to complete this design and its fabrication to the satisfaction of the sponsors and advisors.

5 Conclusion

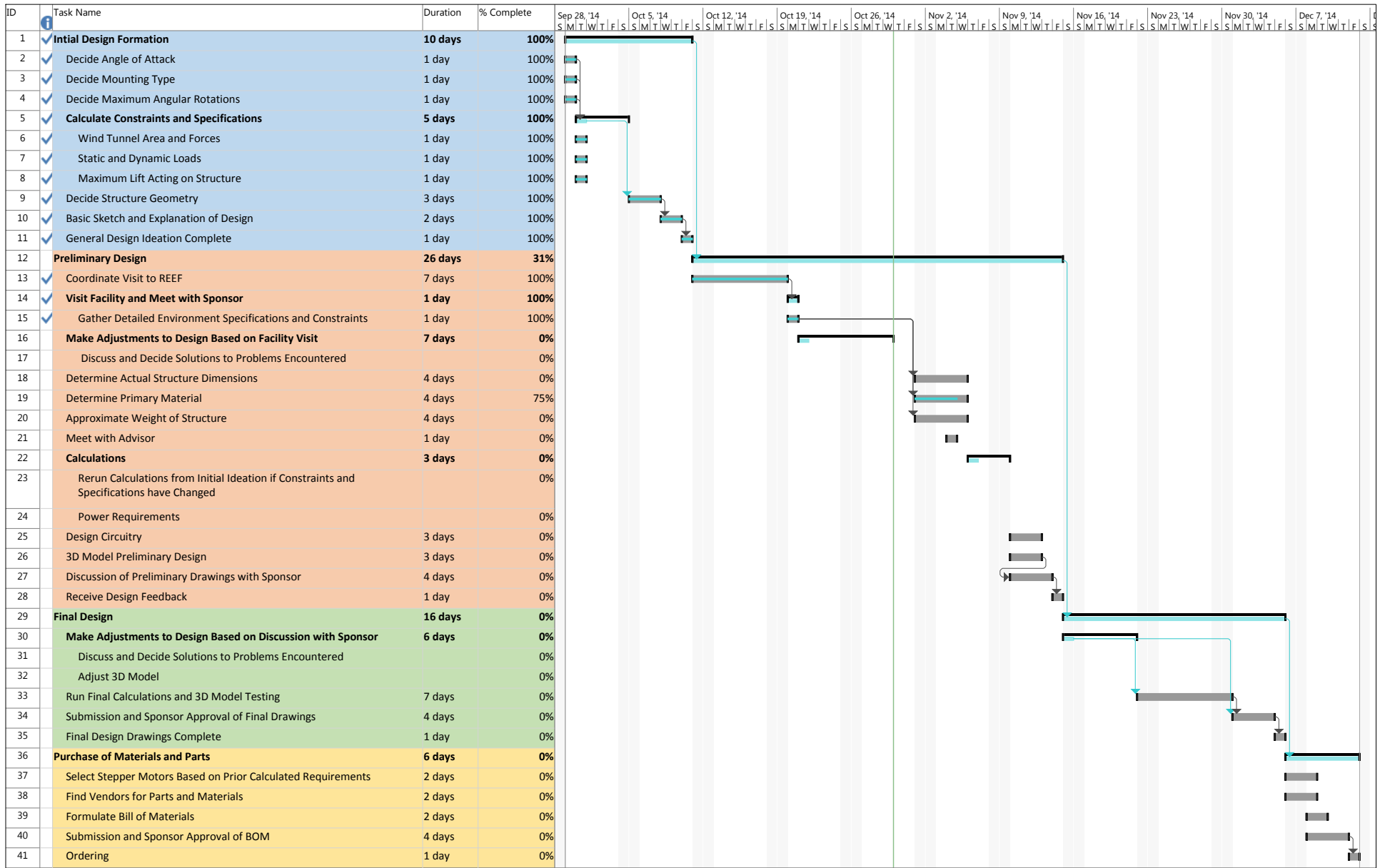
The previous robotic arm used for this wind tunnel was relocated to another research area, and the tunnel can no longer be used to carry out tests without a mounting mechanism. The goal of this project is to creating a mounting mechanism that can adjust the pitch and yaw of the specimen during wind tunnel operation. During operation, the specimen must remain located in the center of the 42 in² test section. The mechanism will also utilize stepper motors with encoders and a servo control unit. Per the sponsor's suggestion, the design will feature a sting mount. This will minimize flow disruption around the test specimen and therefore impact the majority of test results the least.

A visit has been made to the REEF facility by a member of the team to speak with sponsor as well as get more details about the project and specifics of the wind tunnel. The sponsor has decided to construct a base himself for the mounting mechanism, using materials that he previously acquired. This has opened up about \$500 in funds that were previously allocated to purchasing 80/20 material for the base structure. This change was accounted for in the budget shown above (section 2.4) and rough estimate project that the project will come in under the allotted budget. Along with this change in the project parameters the sponsor sent back materials to be utilized in the project. Materials sent back were position servo motor, 3 data acquisition cards, stepper motor with encoder, and data acquisition controller.

The next portion of the project will be to finalize a design and drawings so that materials may be procured for the construction of a prototype mounting mechanism. A meeting with the faculty advisor has been scheduled for the upcoming week in which these details will be discussed. Dr. Kumar has produced and utilized arc based systems to change the pitch of a model so his experience and input will be very beneficial in completing the design.

6 References

- 1) Dunbar, Brian. "Unitary Plan Wind Tunnel 11-by 11-foot Transonic Test Section." NASA. NASA, n.d. Web. 23 Sept. 2014.
- 2) Benson, Tom. "Model Mounts." Model Mounts. NASA, 12 June 2014. Web. 23 Sept. 2014.
- 3) Benson, Tom. "Internal Force Balance." Internal Force Balance. NASA, 12 June 2014. Web. 23 Sept. 2014.
- 4) Benson, Tom. "External Force Balance." External Force Balance. NASA, 12 June 2014. Web. 23 Sept. 2014.



Project: GanttChart

Task	Project Summary	Manual Task	Start-only	Deadline
Split	Inactive Task	Duration-only	Finish-only	Progress
Milestone	Inactive Milestone	Manual Summary Rollup	External Tasks	Manual Progress
Summary	Inactive Summary	Manual Summary	External Milestone	Progress

Givens

Design Properties

$$\text{radius_arc} := 50\text{in}$$

$$L_sting := 50\text{in}$$

$$\rho_{\text{air}} := 1.2 \frac{\text{kg}}{\text{m}^3}$$

$$A_{\text{max}} := 2\text{ft} \cdot 1\text{ft} = 0.186\text{m}^2$$

$$V_{\text{max}} := 22 \frac{\text{m}}{\text{s}}$$

$$\alpha_{\text{max}} := 30\text{deg}$$

$$\beta_{\text{max}} := 30\text{deg}$$

Aluminum Properties

$$\rho_{\text{alum}} := 2.7 \cdot 10^{-3} \frac{\text{kg}}{\text{mL}} = 2.7 \times 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$UTS_{\text{alum}} := 45\text{ksi} = 3.103 \times 10^8 \text{Pa}$$

$$USS := 29.7\text{ksi} = 2.048 \times 10^8 \text{Pa}$$

$$YS := 39.9\text{ksi} = 2.751 \times 10^8 \text{Pa}$$

$$E := 10000\text{ksi} = 6.895 \times 10^{10} \text{Pa}$$

$$G := 3770\text{ksi} = 2.599 \times 10^{10} \text{Pa}$$

Force and Moment Calculations

$$F_{x_drag} := \frac{1}{2} \cdot \rho_{\text{air}} \cdot V_{\text{max}}^2 \cdot A_{\text{max}} \cdot \sin(\alpha_{\text{max}}) = 26.979\text{N}$$

$$F_{y_drag} := \frac{1}{2} \cdot \rho_{\text{air}} \cdot V_{\text{max}}^2 \cdot A_{\text{max}} \cdot \cos(\alpha_{\text{max}}) = 46.729\text{N}$$

$$\text{Moment}_{\text{max}} := F_{x_drag} \cdot \text{radius_arc} = 34.263\text{N}\cdot\text{m}$$

$$\text{Torque}_{\text{max}} := L_{\text{sting}} \cdot \cos(\beta_{\text{max}}) \cdot F_{x_drag} = 29.673\text{N}\cdot\text{m}$$

$$\text{Motor_torque} := \text{Moment}_{\text{max}}$$

$$F_{\text{lift}} := 5\text{N} \quad \text{random assumption...}$$

Structural Calculations

-Assuming full aluminum rods as structure base

$$\text{Shear_force_arm} := Fx_drag = 26.979 \text{ N}$$

$$d_min_shear := \sqrt{\frac{3 \cdot \text{Shear_force_arm}}{\pi \cdot \text{USS}}} = 0.014 \cdot \text{in}$$

-Bar diameter must be greater than 0.014 inches if only shear is considered

$$d_min_bend := \sqrt[3]{64 \cdot \frac{\text{Moment_max}}{\pi \cdot \text{YS}}} = 0.537 \cdot \text{in}$$

-Shear diameter is larger, so minimum diameter is 0.537m

$$\text{FOS} := 5$$

-Select quarter inch bar as material diameter

$$d_min := 1.5 \text{in} = 0.038 \cdot \text{m}$$

$$\text{mass_arc} := \frac{\pi}{4} \cdot d_min^2 \cdot \left(2 \cdot \pi \cdot \text{radius_arc} \cdot \frac{115}{360} \cdot \rho_{\text{alum}} \right) = 17.299 \cdot \text{lb}$$

$$\text{mass_sting} := \frac{\pi}{4} \cdot d_min^2 \cdot \text{radius_arc} \cdot \rho_{\text{alum}} = 8.619 \cdot \text{lb}$$

$$\text{mass_extra} := 10 \text{kg}$$

$$\text{mass_total} := \text{mass_arc} + \text{mass_extra} + \text{mass_sting} = 47.964 \cdot \text{lb}$$

$$\text{bearing_normal_force} := \text{mass_arc} \cdot g = 76.949 \text{ N}$$

$$I := \frac{\pi}{4} \cdot \left(\frac{d_min}{2} \right)^4$$

$$\sigma_{\text{bend}} := \text{Moment_max} \cdot \frac{\frac{d_min}{2}}{I} = 6.31 \times 10^6 \text{ Pa}$$

$$\tau_{xy} := \frac{4}{3} \cdot \frac{\text{Shear_force_arm}}{\frac{\pi}{4} \cdot d_min^2} = 3.155 \times 10^4 \text{ Pa}$$

$$\sigma_y := \frac{\text{mass_total} \cdot g}{\left(\frac{\pi}{4}\right) \cdot d_{\text{min}}^2} - \frac{F_{\text{lift}}}{\frac{\pi}{4} \cdot d_{\text{min}}^2} + \sigma_{\text{bend}} = 6.493 \times 10^6 \text{ Pa}$$

$$\sigma_1 := \frac{\sigma_y}{2} + \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 6.493 \times 10^6 \text{ Pa}$$

$$\sigma_2 := \frac{\sigma_y}{2} - \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = -153.316 \text{ Pa}$$

$$\tau_{\text{max}} := \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3.247 \times 10^6 \text{ Pa}$$